Math 10A
Quiz 10; Friday, 8/3/2018
Time: 3 PM
Instructor: Roy Zhao **Name:** ____________________

Circle True or False. (1 point for correct answer, 0 if incorrect)

1. **TRUE** False If two vectors are perpendicular to each other (they form an angle of $90°$), then their dot product is 0.

> **Solution:** The dot product is $\vec{v} \bullet \vec{w} = |v||w|\cos(\theta)$ but $\theta = 90°$ so $\cos\theta = 0$

2. **TRUE** False If we find two different solutions to $A\vec{x} = \vec{b}$, then $|A| = 0$.

> **Solution:** The number of solutions is $0, 1, \infty$. Since there are at least two solutions, there are not 0 or 1 so there must be ∞ solutions so $|A| = 0$.

Show your work and justify your answers. Please circle or box your final answer.

3. (10 points) (a) (2 points) Let $A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & -1 & 2 \\ 1 & 1 & 1 \end{pmatrix}$ and $\vec{v} = \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}$. Calculate $A\vec{v}$.

> **Solution:**
> $$A\vec{v} = \begin{pmatrix} 3 \\ 2 \\ 4 \end{pmatrix}$$

(b) (6 points) Calculate A^{-1} using Gaussian elimination.

> **Solution:** Using Gaussian elimination
> $$\left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 2 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{array}\right) \xrightarrow{III-I} \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 2 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right)$$
> $$\xrightarrow{I+II} \left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 1 & 0 \\ 0 & -1 & 2 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right) \xrightarrow{I-2III,II-2III} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & 1 & -2 \\ 0 & -1 & 0 & 2 & 1 & -2 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{II\cdot(-1)} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & 1 & -2 \\ 0 & 1 & 0 & -2 & -1 & 2 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right)$$

So the inverse is $\begin{pmatrix} 3 & 1 & -2 \\ -2 & -1 & 2 \\ -1 & 0 & 1 \end{pmatrix}$.

(c) (2 points) Find $\vec{x}$ such that $A\vec{x} = \vec{v}$. (Hint: Use your answer from above)

Solution:

$$\vec{x} = A^{-1}\vec{v} = \begin{pmatrix} 7 \\ -4 \\ -2 \end{pmatrix}$$